CSIR DECEMBER 2015 QUESTION AND SOLUTION

PART 'A'

1. A circle drawn in the *x-y* coordinate plane passes through the origin and has chords of lengths 8 units and 7 units on the *x* and *y* axes, respectively. The coordinates of its centre are

(a) (8, 7) (b) (-8, 7)
(c) (-4, 3.5) (d) (4, 3.5)

2. The probability that a ticketless traveler is caught during a trip is 0.1. If the traveler makes 4 trips, the probability that he/she will be caught during at least one of the trips is:

(a) $1-(0.9)^4$ (b) $(1-0.9)^4$
(c) $1-(1-0.9)^4$ (d) $(0.9)^4$

3. The statement: "The father of my son is the only child of your parents"

(a) can never be true
(b) is true in only one type of relation
(c) can be true for more than one type of relations
(d) can be true only in a polygamous family

4. The base diameter of a glass is 20% smaller than the diameter at the rim.The glass is filled to half th height. The ratio of empty to filled volume of the glass is

(a) $\frac{\sqrt{10}-\sqrt{9}}{\sqrt{9}-\sqrt{8}}$ (b) $\frac{10-9}{9-8}$
(c) $\frac{10^2-9^2}{9-8}$ (d) $\frac{10^3-9^3}{9^3-8^3}$

5. The number of diagonals of a convex deodecagon (12-gon) is

(a) 66 (b) 54
(c) 55 (d) 60

6. One is required to tile a plane with congruent regular polygons. With which of the following polygons is this possible?

(a) 6-gon (b) 8-gon
(c) 10-gon (d) 12-gon

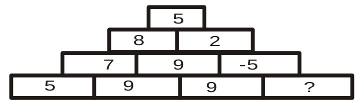
7. Suppose three meetings of a group of professors where arranged in Mumbai, Delhi and Chennai. Each professor of the group attended exactly two meetings. 21 professors attended Mumbai meeting, 27 attended Delhi meeting and 30 attended Chennai meeting. How Many of them attended both the Chennai and Delhi meeting

(a) 18
(b) 24
(c) 26
(d) Cannot be found from the above information

8. A wheel barrow with unit spacing between its wheels is pushed along a semi-circular path of mean radius 10. The difference between distances covered by the inner and outer wheels is

(a) 0 (b) 10
(c) π (d) 2π

9. The missing number is



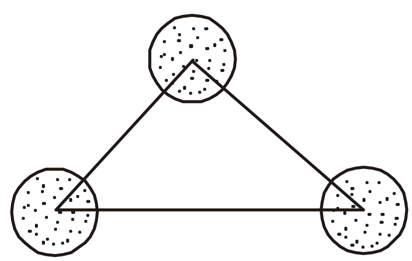
(a) -19 (b) -5
(c) 9 (d) -9

10. Decode

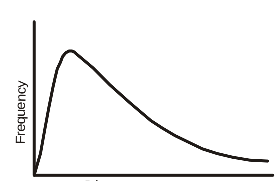
G	E	N	T	S	T	U
I	S	S	O	L	V	D
L	I	I	S	P	A	E
L	M	H	T	R	B	N
E	E	L	B	O	L	T
T	N	I	Y	B	E	S

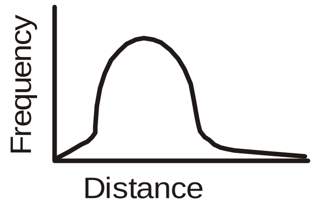
(A) GENT STUDENT CAUSE LITTLE HEART BURNS
(B) STUDENTS ARE INTELLIGENT BUT PROBLEM IS NOT SOLVABLE
(C) THIS PROBLEM IS UNSOLVABLE BY ANY STUDENT
(D) THIS PROBLEM IS SOLVABL BY INTELLIGENT STUDENTS

11. Three circles of equal diameters are placed such that their centres make an equilateral triangle as in the figure



Within each circle, 50 points are randomly scattered. The frequency distribution of distances between all possible pairs of points will look as

1. 

2. 

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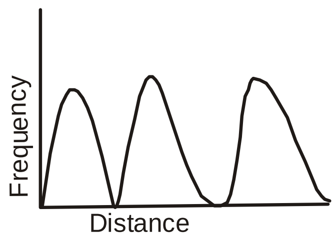
4. 

12. How many digits are there in 3^{16} when it is expressed in the decimal form?

(a) Three (b) Six

(c) Seven (d) Eight

13. Most Indian tropical fruit trees produce fruits in April-May. The best possible explanation for this is

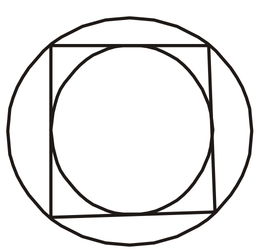
(a) optimum water availability for fruit production

(b) The heat allows quicker ripening of fruit.

(c) animals have no other source of food in summer.

(d) the impending monsoon provides optimum conditions for propagation.

14. There is an inner circle and an outer circle around a square. What is the ratio of the area of the outer circle to that of the inner circle?



(a) $\sqrt{2}$ (b) 2

(c) $2\sqrt{2}$ (d) $\sqrt{3/2}$

15. Write d = 1 degree r = 1 radian and g =1 grad. Then which of the following is true? (100 grad = a right - angle)

(a) cos d < cos r < cos g

(b) cos r < cos g , cos d

(c) cos r < cos d < cos g

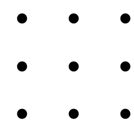
(d) cos g < cos d < cos r

16. A vendor sells articles having a cost price of Rs. 100 each .He sells there articles at a premium price during first eight months, and at a sale price which is half of the premimum price during next four months He makes a net profit of 20% at the end of the year. Asuming that equal nmbers of articles are sold each month what is the premimum price of the article?

(a) 122 (b) 144

(c) 150 (d) 160

17.



The minimum number of straight lines required to connect the nine points above without lifting the pen or retracing is

(a) 3 (b) 4

(c) 5 (d) 6

18. "The clue is hidden in this statement" real the note handed to Sherlock by Moriarty, who hid the stolen treasure in one of the ten pillars. Which pillar is it?

(a) X (b) II

(c) III (d) IX

19. Three boxes are coloured red, blue and green and so are three balls. In how many ways can one put the balls one in each box such that no ball goes into the box of its own colur?

(a) 1 (b) 2

(c) 3 (d) 4

20. Let A, B be the ends of the longest diagonal of the unit cube. The length of the shortest path from A to B along the surface is

(a) $\sqrt{3}$ (b) $1+\sqrt{2}$

(C) $\sqrt{5}$ (D) 3

PART 'B'

21. If A is a 5×5 real matrix with trace 15 and if 2 and 3 are eigenvalues of A, each with algebraic multiplicity 2, then the determinant of A is equal to

(a) 0 (b) 24

(c) 120 (d) 180

22. Let $f:\mathbb{R}\to\mathbb{R}$ be a twice continuously differentiable function, with $f(0)=f(1)=f'(0)=0$. Then

(a) f'' is the zero function.

(b) $f''(0)$ is zero

(c) $f''(x)=0$ for some $x\in(0,1)$.

(d) f'' never vanishes

23. Let $S_n=\sum_{k=1}^{n}\frac{1}{k}$. Which of the following is true?

(a) $S_{2^n} \geq \frac{n}{2}$ for every $n \geq 1$.

(b) S_n is bounded sequence.

(c) $|S_{2^n} - S_{2^{n-1}}| \to 0$ as $n \to \infty$.

(d) $\frac{S_n}{n} \to 1$ as $n \to \infty$.

24. For a positive integer n, let P_n denote the vector space of polynomials in one variable x with real co-efficients and with degree $\leq n$. Consider the map $T : P_2 \to P_4$ defined by $T(p(x)) = p(x^2)$. . Then

(a) T is a linear transformation and dim range(T) = 5

(b) T is a linear transformation and dim range(T) = 3.

(c) T is a linear transformation and dim range(T) = 2.

(d) T is not a linear transformation.

25. Let A be a real 3×4 matrix of rank 2. Then the rank of $A^t A$, where A^t denotes the transpose A, is

(A) exactly 2

(b) exactly 3

(c) exactly 4

(d) at most 2 but not necessarily 2

26. Let S denote the set of all the prime numbers p with the property that the matrix $\begin{bmatrix} 91 & 31 & 0 \\ 29 & 31 & 0 \\ 79 & 23 & 59 \end{bmatrix}$ has an inverse in the field $\mathbb{Z}/p\mathbb{Z}$. Then

(a) S = {31} (b) S = {31, 59}

(c) S = {7, 13, 59} (d) S is infinite

27. Let $f : [0, \infty) \to [0, \infty)$ be a continuous function. Which of the following is correct?

(a) There is $x_0 \in [0, \infty)$ such that $f(x_0) = x_0$.

(b) If $f(x) \leq M$ for all $x \in [0, \infty)$ for some M > 0, then there exists $x_0 \in [0, \infty)$ such that $f(x_0) = x_0$

(c) If f has a fixed point then it must be unique

(d) f does not have a fixed point unless it is differentiable on (0, ∞)

28. Consider the quadratic form $Q(v) = v^t Av$, where

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, v = (x, y, z, w)$$. Then

(a) Q has rank 3

(b) $xy + z^2 = Q(Pv)$ for some invertible 4×4 real matrix P.

(c) $xy + y^2 + z^2 = Q(Pv)$ for some invertible 4×4 real matrix P.

(d) $x^2 + y^2 - zw = Q(Pv)$ for some invertible 4×4 real matrix P.

29. Let $A \neq I_n$ be an $n \times n$ matrix such that $A^2 = A$, where I_n is the identity matrix of order n. Which of the following statements is false?

(a) $(I_n - A)^2 = I_n - A$

(b) Trace (A) = Rank (A).

(c) Rank (A) + Rank $(I_n - A) = n$.

(d) The eigenvalues of A are each equal to 1.

30. For $(x, y) \in \mathbb{R}^2$ with $(x, y) \neq (0, 0)$, let $\theta = \theta\ (x, y)$ be the unique real number such that $-\pi < \theta \leq \pi$ and (x, y) = $(r\cos\theta, r\sin\theta)$ where $r = \sqrt{x^2 + y^2}$. Then the resulting function $\theta : \mathbb{R}^2 \setminus \{(0,0)\} \to \mathbb{R}$ is

(a) differentiable

(b) continuous, but not differentiable

(c) bounded but not continuous

(d) neither bounded, nor continuous.

31. $\lim_{n\to\infty} \frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{2}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{6}} + \ldots + \frac{1}{\sqrt{2n}+\sqrt{2n+2}}\right)$ is

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\sqrt{2}+1$ (d) $\frac{1}{\sqrt{2}+1}$

32. Let A be a closed subset of $\mathbb{R}$, $A \neq \phi$, $A \neq \mathbb{R}$. Then A is

(a) The closure of the interior of A

(b) a countable set.

(c) a compact set

(d) not open

33. A group G is generated by the elements x, y with the relations $x^3 = y^2 = (xy)^2 = 1$. The order of G is

(a) 4 (b) 6

(c) 8 (d) 12

34. Let $a, b, c, d \in \mathbb{R}$ be such that $ad - bc > 0$. Consider the Mobius transformation $T_{a,b,c,d}(z) = \frac{az+b}{cz+d}$. Define $H_+ = \{z \in \mathbb{C} : \text{Im}(z) > 0\}$, $H_- = \{z \in \mathbb{C} : \text{Im}(z) < 0\}$, $R_+ = \{z \in \mathbb{C} : \text{Re}(z) > 0\}$, $R_- = \{z \in \mathbb{C} : \text{Re}(z) < 0\}$.

Then $T_{a,b,c,d}$ maps

(a) H_+ to H_+

(b) H_+ to H_-

(c) R_+ to R_+

(d) R_+ to R_-

35. What is the total number of positive integer solutions to the equation

$(x_1+x_2+x_3)\ (y_1+y_2+y_3+y_4)=15$?

(a) 1 (b) 2

(c) 3 (d) 4

36. Which of the following is an irreducible factor of $x^{12}-1$ over $\mathbb{Q}$?

(a) $x^8+x^4+|1$.

(b) x^4+1.

(c) x^4-x^2+1.

(d) $x^5-x^4+x^3-x^2+x-1$.

37. What is the cardinality of the set $\{z\in\mathbb{C}\ |\ z^{98}=1$ and $z^n\neq 1$ for any $0<n<98\}$?

(a) 0 (b) 12

(c) 42 (d) 49

38. For a subset A of the topological space X, let $\hat{A}$ denote the union of the set A and all those connected components of $X\setminus A$ which are relatively compact inX (i.e the closure is compact). Then for every $A\subseteq X$.

(a) A is compact (b) $\hat{A}=\hat{\hat{A}}$,

(c) A is connected (d) $\hat{A}=X$.

39. Let R be a Euclidean domain such that R is not a field. Then the polynomial ring R[X] is always

(a) a Euclidean domain

(b) a principal ideal domain but not a Euclidean domain

(c) a unique factorization domain, but not a principal ideal domain

(d) not a unique factorization domain

40. Consider the following power series in the complex variable z.

$f(z)=\sum_{n=1}^{\infty} n\log n\ z^n,\ g(x)=\sum_{n=1}^{\infty}\frac{e^{n^2}}{n}z^n$. If r, R are the radii of convergence of f and g respectively then

(a) $r=0,\ R=1$. (b) $r=1,\ R=0$

(c) $r=1,\ R=\infty$ (d) $r=\infty,\ R=1$.

41. A force $5\hat{i}-2\hat{j}+3\hat{k}$ acts on a particle with position vector $2\hat{i}+\hat{j}-2\hat{k}$. The torque of the force about the origin is

(a) $\hat{i}+16\hat{j}+9\hat{k}$ (b) $-\hat{i}-16\hat{j}-9\hat{k}$

(c) $\hat{i}+16\hat{j}-9\hat{k}$ (c) $\hat{i}-16\hat{j}+9\hat{k}$

42. The functional

$$I(y(x))=\int_a^b (y^2+y'^2-2y\sin x)dx\ ,$$

has the following extremal with c_1 and c_2 as arbitrary constatnts.

(a) $y=C_1e^{2x}+C_2e^{-2x}+\frac{1}{2}\sin x$

(b) $y=C_1e^{x}+C_2e^{-x}+\frac{1}{2}\sin x$

(c) $y=C_1e^{x}+C_2e^{-x}-\frac{1}{2}\sin x$

(d) $y=C_1e^{2x}+C_2e^{-2x}+\frac{1}{2}\cos x$

43. The resolvent kernel $R(x,t,\lambda)$ for the Volterra integral equation $\varphi(x)=x+\lambda\int_a^x\varphi(s)ds$, is

(a) $e^{\lambda(x+t)}$ (b) $e^{\lambda(x-t)}$

(c) $\lambda e^{(x+t)}$ (d) $e^{\lambda xt}$

44. The solution of the initial value problem

$$(x-y)\frac{\partial u}{\partial x}+(y-x-u)\frac{\partial u}{\partial y}=u,$$

$u(x,0)=1,$ satisfies

(a) $u^2(x-y+u)+(y-x-u)=0$.

(b) $u^2(x+y+u)+(y-x-u)=0$.

(c) $u^2(x-y+u)-(y+x+u)=0$.

(d) $u^2(x-y+u)+(y+x-u)=0$.

45. Let $f(x)=ax+100$ for $a\in\mathbb{R}$. then the iteration $x_{n+1}=f(x_n)$ for $n\geq 0$ and $x_0=0$ converges for

(a) $a\ =\ 5$ (b) $a\ =\ 1$

(c) $a\ =\ 0.1$ (d) $a\ =\ 10$

46. The PDE $\frac{\partial^2 u}{\partial x^2}+2\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 u}{\partial y^2}=x.$ has

(a) only one particular integral

(b) a particular integral which is linear in x and y

(c) a particular integral which is a quadratic polynomial is x and y.

(d) more than one particular integral.

47. Consider the ODE on $\mathbb{R}$ $y'(x)=f(y(x))$. If f is an even function and y is an odd function, then

(A) $-y(-x)$ is also a solution

(b) $y(-x)$ is also a solution

(c) $-y(x)$ is also solution

(d) $y(x)y(-x)$ is also a solution

48. Consider the system of ODE in

$$\mathbb{R}^2,\frac{dY}{dt}\ =\ AY, Y(0)=\begin{pmatrix}0\\1\end{pmatrix},\ t>0$$

where $A=\begin{bmatrix}-1 & 1\\0 & -1\end{bmatrix}$ and $Y(t)=\begin{pmatrix}y_1(t)\\y_2(t)\end{pmatrix}$. Then

(a) $y_1(x)$ and $y_2(t)$ are monotonically increasing for $t>0$

(b) $y_1(t)$ and $y_2(t)$ are monotonically increasing for $t>1$.

(c) $y_1(t)$ and $y_2(t)$ are monotonically decreasing for $t > 0$

(d) $y_1(t)$ and $y_2(t)$ are monotonically decreasing for $t > 1$

49. From the six letters A, B, C, D, E and F, three letters are chosen at random with replacement. What is the probability that either the word BAD or the word CAD can be formed from the chosen letters?

(a) $\frac{1}{216}$ (b) $\frac{3}{216}$

(c) $\frac{6}{216}$ (d) $\frac{12}{216}$

50. Let X be a radom variable which is symmetric about 0. Let F be the cumulative distribution function of X. Which of the following statements is always true?

(a) F(x) + F(-x) = 1 for all $x \in \mathbb{R}$.

(b) F(x) - F(-x) = 0 for all $x \in \mathbb{R}$.

(c) F(x) + F(-x) = 1 + P(X=x) for all $x \in \mathbb{R}$.

(d) F(x) + F(-x) = 1 -P(X=-x) for all $x \in \mathbb{R}$.

51. A set of *N* observations resulted in k distinct values $x_1, x_2, ..., x_k$ with respective frequencies $f_1, f_2, ... f_k$ so that $\sum_{i=1}^{k} f_i = N$ Another *k* observations resulted in observations $x_1, x_2, ..., x_k$ once each so that the modified (new) sample of size N+k has observation x_i with frequency $f_i + 1$

(a) The new mean is necesarily less than or equal to the original mean.

(b) The new median is necessarily more than or equal to the original median.

(c) The new variance is necessarily less than or equal to the origion variance

(d) The new mode will be same as the original mode.

52. Let Y_1, Y_2, Y_3 and Y_4 be uncorrelated observations with common unknown variance σ^2 and expectations given by

$E(Y_1) = \beta_1 + \beta_2 + \beta_3 = E(Y_2),$

$E(Y_3) = \beta_1 - \beta_2 = E(Y_4)$

where β_1, β_2 and β_3 are unknown parameters.

Define $e_1 = \frac{1}{\sqrt{2}}(Y_1 - Y_2)$ and $e_2 = \frac{1}{\sqrt{2}}(Y_3 - Y_4)$ An unbiased estimator of σ^2 is

(a) $\frac{1}{2}(e_1^2 - e_2^2).$ (b) $\frac{1}{2}(e_1^2 + e_2^2).$

(c) $\frac{1}{4}(e_1^2 + e_2^2).$ (d) $e_1^2 + e_2^2.$

53. Let X_1, X_2, X_3 and X_4 independent and identically distributed random variables with common distribution normal with mean μ and variance 2. If the prior distribution of μ is normal with mean 0 and variance $\frac{1}{2}$, then which of the following is true?

(a) The prior distribution is not a conjugate prior

(b) Posterior mode μ given X_1, X_2, X_3 and X_4 is $\frac{\sum_{i=1}^{4} X_i}{8}$

(c) Posterior median of μ given X_1, X_2, X_3 and X_4 is $\frac{\sum_{i=1}^{4} X_i}{4}$

(d) Posterior variance of μ given X_1, X_2, X_3 and X_4 is $\left(\frac{\sum_{i=1}^{4} X_i}{4}\right)$

54. Let the $n \times 1$ vector $\underline{x}$ follow an *n*-variate normal distribution with mean vector $\underline{\mu}(\neq \underline{0})$ and variance covariance matrix $V(\neq I_n$, the n^{th} order identity matrix). Also let A be a symmetric matrix of order *n*. Which of the following statements is true?

(a) $\underline{x}' A \underline{x}$ follows a central chi-square distribution if an only if $(AV)^2 = AV$

(b) $\underline{x}' A \underline{x}$ follows a central chi-square distribution if and only if $A^2 = A$

(c) The mean of $\underline{x}' A \underline{x}$ is $\underline{\mu}' A \mu + tr(AV)$ where tr(.) denotes the trace of a square matrix.

(d) $\underline{x}' A \underline{x}$ always has a central chi-square distributiopn with *n* degree of freedom

55. Consider the following Linear Programming Problem. Max $x_1 + \frac{5}{2} x_2$ subject to

$$5x_1 + 3x_2 \le 15$$
$$-x_1 + x_2 \le 1$$
$$2x_1 + 5x_2 \le 10$$
$$x_1, \ x_2 \ge 0$$

The problem

(a) Has no feasible solution

(b) has infinitely many optimal solutions

(c) has a unique optimal solution

(d) has an unbounded solution

56. Let X_i' 's be independent random variables such that X_i' s are summetric about 0 and $Var(X_i) = 2i - 1$, for $i \ge 1$. Then, $\lim_{n\to\infty} P(X_1 + X_2 + ... + X_n > n\ long)$

(a) does not exist (b) equals $1/2$

(c) equals 1. (d) equals 0.

57. Consider a randomized block design involving 3 treatments and 3 replicates and let t_i denote the

effect of the i^{th} treatment (i =, 2, 3). If σ^2 denotes the variance of an observation which of the following statements is true?

(a) The variance of the best linear unbiases estimators (BLUE) of $(t_1 - t_2)/\sqrt{2}$ and $(t_1 - 2t_2 + t_3)/\sqrt{6}$ are equal .

(b) The covariance between the BLUE of $t_1 - t_3$ and the BLUE of $t_1 - 2t_2 + t_3$ is $2\sigma^2/3$

(c) The varince of the BLUE of $t_i - t_j, (i \neq j,\ i, j = 1,\ 2,\ 3,\ )$ is $\sigma^2 \backslash 3$

(d) The variance of the BLUE of $(t_1 - 2t_2 + t_3)$ is $\sigma^2/6$

58. Let $Y_1,\ Y_2$ be two independent random variables taking value -1 and +1 with probability $\frac{1}{2}$ each. Define $X_1 = Y_1,\ X_2 = Y_2, X_3 = X_2\ X_1, ..., X_n\ = X_{n-1}X_{n-2}$ for $n \geq 3$. Then

(a) $P(X_8 = 1,\ X_9 = 1,\ X_{10} = -1) = \frac{1}{4}$

(b) $P(X_8 = 1,\ X_9 = 1,\ X_{10} = 1) = \frac{1}{4}$

(c) $P(X_8 = 1,\ X_9 = 1,\ X_{10} = -1) = \frac{1}{8}$

(d) $P(X_8 = 1,\ X_9 = 1,\ X_{10} = 1) = \frac{1}{8}$

59. Let $X_1,\ X_2, ..., X_n$ be a random sample from uniform $(\theta,\ 5\theta),\ \theta > 0.$ Define $X_{(1)} = \min\ \{X_1, X_2, ..., X_n\}$ and $X_{(n)} = \max\{X_1, X_2, ..., X_n\}$. Maximum likelihood estimator of θ is

(a) $\frac{X_{(1)}}{5}$ (b) $X_{(n)}$

(c) $X_{(1)}$ (d) $\frac{X_{(n)}}{5}$

60. Consider the problem of testing $H_0 : X \sim$ Normal with mean 0 and variance $\frac{1}{2}$ against $H_1 : X \sim$ Cauchy $(0,\ 1)$. Then for testing H_0 against $H_1,$ the most powerful size α test

(a) does not exist

(b) rejects H_0 if and only if |x| > c_2 where c_2 is such that the test is of size α

(c) rejects H_0 if an only if |x| < c_3 where c_3 is such that the test is of size α

(d) rejects H_0 if and only if |x| < c_4 or |x| > c_5 , $c_4 < c_5$ where c_4 and c_5 are such that the test is of size α .

PART 'C'

61. Let $p_n(x) = a_n x^2 + b_n x$ be a sequence of quadratic polynomials where $a_n,\ b_n \in \mathbb{R}$ for all $n \geq 1$. Let $\lambda_0,\ \lambda_1$ be distinct nonzero real numbers such that $\lim_{n\to\infty} p_n(\lambda_0)$ and $\lim_{n\to\infty} p_n(\lambda_1)$ exist Then

(a) $\lim_{n\to\infty} p_n(x)$ exists for all $x \in \mathbb{R}$.

(b) $\lim_{n\to\infty} p_n'(x)$ exists for all $x \in \mathbb{R}$.

(c) $\lim_{n\to\infty} p_n \frac{(\lambda_0 + \lambda_1)}{2}$ does not exist.

(d) $\lim_{n\to\infty} p_n' \frac{(\lambda_0 + \lambda_1)}{2}$ does not exist.

62. Which of the following sets are compact?

(a) $\{(x, y, z) \in \mathbb{R}^3\ :\ x^2 + y^2 + z^2 = 1\}$ in the Euclidean topology.

(b) $\{(z_1, z_2, z_3) \in \mathbb{C}^3\ :\ z_1^2 + z_2^2 + z_3^2 = 1\}$ in the Euclidean topology.

(c) $\prod_{n=1}^{\infty} A_n$ with product topology, where $A_n = \{0,\ 1\}$ has discrete topology for n = 1, 2,3,...

(d) $\{z \in \mathbb{C}\ : |\operatorname{Re} z| \leq a\}$ in the Euclidean topology for some fixed positive real number a.

63. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $\sup_{x\in\mathbb{R}} |f'(x)| < \infty.$ Then

(a) f maps a bounded sequence to a bounded sequence.

(b) f maps a Cauchy sequence to a Cauchy sequence.

(c) f maps a convergent sequence to a convergent sequence

(d) f is uniformly continuous.

64. For $(x, y) \in \mathbb{R}^2$ consider the series $\lim_{n\to\infty} \sum_{\ell, k=0}^{n} \frac{k^2 x^k y^\ell}{\ell!}$ Then the series converges for (x, y) in

(a) $(-1,\ 1) \times (0,\ \infty)$ (b) $\mathbb{R} \times (-1,\ 1)$

(c) $(-1,\ 1) \times (-1,\ 1)$ (d) $\mathbb{R} \times \mathbb{R}$

65. Let $f : (0,\ 1) \to \mathbb{R}$ be continuous . Suppose that $|f(x) - f(y)| \leq |\cos x - \cos y|$ of all $x, y \in (0,1)$. Then.

(a) f is discountinuous at least at one point in (0, 1).

(b) f is continuous everywhere on (0, 1) but not unidormly continuous on (0, 1)

(c) f is uniformly continuous on (0, 1)

(d) $\lim_{x\to 0} f(x)$ exists.

66. Let $f:\mathbb{R}^2\to\mathbb{R}^2$ be the function

$$f(r,\ \theta)=(r\cos\theta,\ r\sin\theta).$$

Then for which of the open subsets U of $\mathbb{R}^2$ given below f restricted to U admits an inverse?

(a) $U=\mathbb{R}^2$

(b) $U=\{(x,y)\in\mathbb{R}^2\ :\ x>0,\ y>0\}$

(c) $U=\{(x,y)\in\mathbb{R}^2\ :\ x^2+y^2<1\}$

(d) $U=\{(x,y)\in\mathbb{R}^2\ :\ x<-1,\ y<-1\}$

67. Let $S\subset\mathbb{R}^2$ be defined by

$$S=\{\left(m+\frac{1}{4|p|},n+\frac{1}{4|q|}\right)\ :\ m,n,p,q,\in\mathbb{Z}\}$$

Then

(a) S is discrete in $\mathbb{R}^2$

(b) The set of limit points of S is the set $\{(m,n):m,n\ \in\ \mathbb{Z}\ \}$.

(c) S^c is connected but not path connected

(d) S^c is path connected.

68. Let $A=\{(x,y)\in\mathbb{R}^2\ :\ x+y\neq -1\}$ Define $f:A\to\mathbb{R}^2$ by $f(x,y)=(\frac{y}{1+x+y},\frac{x}{1+x+y})$. Then

(a) the deternimant of the jacobian of f does not vanish on A.

(b) f is infinitely differentiable on A.

(c) f is one to one

(d) f(A) = $\mathbb{R}^2$

69. Let $f:\mathbb{R}^2\to\mathbb{R}^2$ be given by the formula

$$f(x,y)=(3x+2y+y^2+|xy|,\ 2x+3y+x^2+|xy|).$$

Then

(a) f is discontinuous at (0, 0)

(b) f is continuous at (0, 0) but not differentiable at (0, 0)

(c) f is differentiable at (0, 0)

(d) f is differentiable at (0, 0) and the derivative Df(0, 0) is invertiable.

70. Consider all sequences $\{f_n\}$ of real valued continuous function on$[0,\infty)$. Identify which of the following statements are correct.

(a) If $\{f_n\}$ converges to f pointwise on $[0,\ \infty)$, then $\lim_{n\to\infty}\int_0^\infty f_n(x)dx=\int_0^\infty f(x)dx$

(b) If $\{f_n\}$ converges to f unifromly on $[0,\infty)$, then $\lim_{n\to\infty}\int_0^\infty f_n(x)dx=\int_0^\infty f(x)dx$

(c) If $\{f_n\}$ converges to f uniformly on $[0,\ \infty)$ then f is continuou on $[0,\ \infty)$

(d) There exists a sequence of continuous functions $\{f_n\}$ on $[0,\infty)$ such that $\{f_n\}$ converges to f uniforlmy on $[0,\ \infty)$ but $\lim_{n\to\infty}\int_0^\infty f_n(x)dx\neq\int_0^\infty f(x)dx.$

71. Let t and a be positive real numbers Define $B_a=\{x=(x_1,x_2,...,x_n)\in\mathbb{R}^n\,|\,x_1^2+x_2^2+...+x_n^2\ \le\ a^2\}$. Then for any compactly supported continuous function f on $\mathbb{R}^n$ which of the following are correct?

(a) $\int_{B_a} f(tx)dx=\int_{B_{ta}} f(x)t^{-n}dx$

(b) $\int_{B_a} f(tx)dx=\int_{B_{t^n a}} f(x)t\ dx$

(c) $\int_{\mathbb{R}^n} f(x+y)dx=\int_{\mathbb{R}^n} f(x)dx,$ for some $y\in\mathbb{R}^2$

(d) $\int_{\mathbb{R}^n} f(tx)dx=\int_{\mathbb{R}^n} f(x)t^n dx.$

72. Let G_1 and G_2 be two subsets of $\mathbb{R}^2$ and $f:\mathbb{R}^2\to\mathbb{R}^2$ be a function . Then

(a) $f^{-1}(G_1\cup G_2)=f^{-1}(G_1)\cup f^{-1}(G_2)$

(b) $f^{-1}(G_1^c)=(f^{-1}(G_1))^c$

(c) $f(G_1\cap G_2)=f(G_1)\cap f(G_2)$

(d) If G_1 is open and G_2 is closed then $G_1+G_2=\{x+y:x\in G_1,y\in G_2\}$ is neither open nor closed.

73. Let A and B be $n\times n$ matrices over $\mathbb{C}$. Then

(a) AB and BA always have the same set of eigenvalues.

(b) If AB and BA have the same set of eigen values then AB=BA.

(c) If A^{-1} exists then AB and BA are similar.

(d) The rank of AB is always the same as the rank of BA.

74. Let V be a finite dimensional vector space over $\mathbb{R}$. Let $T:V\to V$ be a linear transformation such that rank $(T^2)=rank(T)$. Then,

(a) Kernal $(T^2)=$ kernal (T)

(b) Range $(T^2)=$ = Range (T)

(c) Kernal (T) $\cap$ Range (T) = {0}.

(d) Kernal (T^2) $\cap$ Range $(T^2)=\{0\}$.

75. Let V be the vector space of polynomials over $\mathbb{R}$ of degree less than or equal to n. For $p(x)=a_0+a_1x+...+a_{n1}x^n$ in V, define a linear transformation $T:V\to V$ b $(Tp)(x)=a_n+a_{n-1}x+...+a_0x^n$. Then

(a) T is one to one (b) T is onto.

(c) T is invertible (d) det $T=\pm 1$.

76. Consider the matrices $A=\begin{bmatrix}2&2&1\\0&2&-1\\0&0&3\end{bmatrix}$ and

$$B=\begin{bmatrix}2&1&0\\0&2&0\\0&0&3\end{bmatrix}.\text{ Then}$$

(a) A and B are similar over the field of rational numbers $\mathbb{Q}$.

(b) A is diagonalizable over the field of rational numbers $\mathbb{Q}$.

(c) B is the jordan canonical form of A.

(d) The minimal polynomial and the characteristic polynomial of A are the same.

77. Let A be an $m\times n$ real matrix and $b\in\mathbb{R}^m$ with $b\neq 0$.

(a) The set of all real solutions of Ax=b is a vector sapce.

(b) If u and v are two solutions of $Ax = b$, then $\lambda u+(1-\lambda)v$ is also a solution of Ax = b. for any $\lambda\in\mathbb{R}$.

(c) For any two solutions u and v of Ax=b, the linear combination $\lambda u+(1-\lambda)v$ is also a solution of Ax = b only when $0\le\lambda\le 1$.

(d) If rank of A is n, then $Ax=b$ has at most one solution

78. Let A be an $n\times n$ matrix over $\mathbb{C}$ such that every nonzero vector of $\mathbb{C}^n$ is an eigenvector of A. Then

(a) All eigenvalues of A are equal

(b) All eigenvalues of A are distinct.

(c) $A=\lambda I$ for some $\lambda\in\mathbb{C}$ where I is then $n\times n$ identity matrix.

(d) If χ_A and m_A denote the characteristic polynomial and the minimal ploynomial respectively then $\chi_A=m_A$

79. Which of the following statements is /are true?

(a) There exists a continuous map $f:\mathbb{R}\to\mathbb{R}$ such that $f(\mathbb{R})=\mathbb{Q}$.

(b) There exists a continuous map $f:\mathbb{R}\to\mathbb{R}$ such that $f(\mathbb{R})=\mathbb{Z}$

(c) There exists a continuous map $f:\mathbb{R}\to\mathbb{R}^2$ such that $f(\mathbb{R})=\{(x,\ y)\in\ \mathbb{R}^2:x^2+y^2=1\}$.

(d) There exists a continuous map $f:[0,1]\cup[2,3]\to\{0,1\}$.

80. Which of the following intervals contains an integer satisfying the following three congruences:

$x\equiv 2(\text{mod } 5)$, $x\equiv 3(\text{mod } 7)$ and $x\equiv 4(\text{mod } 11)$.

(a)	[401, 600]	(b)	[601, 800]
(c)	[801, 10000]	(d)	[1001, 12000]

81. Let a_n denote the number of those permutation σ on {1, 2,...,n} such that σ is a product of exactly two disjoint cycles Then.

(a)	$a_5=50$	(b)	$a_4=14$
(c)	$a_5=40$	(d)	$a_4=11$

82. Let A denote the quotient ring $\mathbb{Q}[X]/(X^3)$. Then

(a) There are exactly three distinct proper ideals in A

(b) There is only one prime ideal in A.

(c) A is an integral domain

(d) Let f,g be in $\mathbb{Q}[X]$ such that $\overline{f}\bullet\overline{g}=0$ in A. Here $\overline{f}$ and $\overline{g}$ denote the image of f and g respectively in A. Then $f(0)$. $g(0) = 0$.

83. Let $\omega=\cos\frac{2\pi}{10}+i\ \sin\frac{2\pi}{10}$. Let $K=\mathbb{Q}(\varepsilon^2)$ and let $L=\mathbb{Q}(\omega^2)$. Then

(a)	$[L:\mathbb{Q}]=10$	(b)	$[L:K]=2$
(c)	$[K:\mathbb{Q}]=4$	(d)	L = K

84. Let G be an simple group of order 60. Then

(a) G has six Sylow -5 subgroups

(b) G has four Sylow-3 subgroups

(c) G has a cyclic subgroup of order 6.

(d) G has a unique element of order 2.

85. Which of the following quotient ring are fields?

(a) $F_3[X]/(X^3+X+1)$, where F_3 is the finite field with 3 element

(a) $\mathbb{Z}[X]/(X-3)$

(c) $\mathbb{Q}[X]/(X^2+X+1)$

(d) $F_2[X]/(X^2+X+1)$ F_2 is the finite field with 2 elements.

86. Let f be an anlytic function in $\mathbb{C}$ Then f is constant if the zero set of f contains the sequence

(a) $a_n=1/n$

(b) $a_n=(-1)^{n-1\frac{1}{n}}$

(c) $a_n=\frac{1}{2n}$

(d) $a_n=n$ if 4 does not divide n and $a_n=\frac{1}{n}$ if 4 divides n

87. For $n\ge 1$ let $(\mathbb{Z}/n\mathbb{Z})^*$ be the group of units of $(\mathbb{Z}/n\mathbb{Z})$. Which of the following groups are cyclic

(a)	$(\mathbb{Z}/10\mathbb{Z})^*$	(b)	$(\mathbb{Z}/2^3\mathbb{Z})^*$
(c)	$(\mathbb{Z}/100\mathbb{Z})^*$	(d)	$(\mathbb{Z}/163\mathbb{Z})^*$

88. Let $f(z)=\frac{1}{e^{n-1}}$ for all $z\in\mathbb{C}$ such that $e^z\neq 1$. Then

(a) f is meromorphic

(b) the only singularities of f are poles

(c) f has infinitely many poles on the imaginary exis.

(d) Each pole of f is simple.

89. Consider the function $f(z)=\frac{1}{z}$ on the annulus

$A = \{z \in \mathbb{C} : \frac{1}{2} < |z| < 2\}$. Which of the following is/are true?

(a) There is a sequence $\{p_n(z)\}$ of polynomials that approximate $f(z)$ uniformly on compact subset of A.

(b) There is a sequence $\{r_n(z)\}$ of rational function whose poles are contained in $\mathbb{C}/A$ and which approximates f(z) uniformly on compact subsets of A.

(c) No sequence $\{p_n(z)\}$ of polynomials approximate $f(z)$ uniformly on compact subsets A.

(d) No sequence $\{r_n(z)\}$ of rational funciton whose poles are contained in $\mathbb{C}$/A, approximate $f(z)$ uniformly on compact subsets of A.

99. Let $C(\mathbb{C})$ denote the vector sapce of continuous complex valued functions on $\mathbb{C}$ and H($\mathbb{C}$) denote the vector sapce of entire functions. For any function f in C($\mathbb{C}$) or H($\mathbb{C}$), and for nay compact subset K of $\mathbb{C}$, define. $\|f\|_k = \sup_{z\in K} |f(z)|$. Then

(a) $\|\bullet\|_K$ is a norm on $C(\mathbb{C})$ for every compact $K \subseteq \mathbb{C}$.

(b) $\|\bullet\|_K$ is an norm on $H(\mathbb{C})$ for every compact $K \subseteq \mathbb{C}$.

(c) $\|\bullet\|_K$ is a norm on $C(\mathbb{C})$ for every compact $K \subseteq \mathbb{C}$ with non-empty interior.

(d) $\|\bullet\|_K$ is a norm on $H(\mathbb{C})$ for every compact $K \subseteq \mathbb{C}$ with non-empty interior

91. Let $y:[0, \infty) \to [0, \infty)$ be a continuously differentiable function satisfying $y(t) = y(0) + \int_0^t y(s)ds$ for $t \geq 0$. Then

(a) $y^2(t) = y^2(0) + \int_0^t y^2(s)ds.$

(b) $y^2(t) = y^2(0) + 2\int_0^t y^2(s)ds.$

(c) $y^2(t) = y^2(0) + 2\int_0^t y(s)ds.$

(d) $y^2(t) = y^2(0) + (\int_0^t y(s)ds). + 2y(0)\int_0^t y(s)ds.$

92. Consider the hamiltonian (H) and the lagrangian (L) for a free particle of mass m and velocity v. Then

(a) H and L are independent of each other

(b) H and L are related but have different dependence on v.

(c) H and L are equal.

(d) Both H and L are quadratic in v.

93. Let u(t) be a continuously differentiable function taking nonnegative values for t > 0 and satisfying $u'(t) = 4u^{3/4}(t); u(0) = 0.$ Then

(a) $u(t) = 0$

(b) $u(t) = t^4$

(c) $u(t) = \begin{cases} 0 & for\ 0 < t < 1 \\ (t-1)^4 & for\ \ t \geq 1. \end{cases}$

(d) $u(t) = \begin{cases} 0 & for\ 0 < t < 10 \\ (t-10)^4 & for\ \ t \geq 10. \end{cases}$

94. Consider a mass m moving in an inverse square central force with characteristic coefficient μ and described by the Lagragian :

$L(r, r, \theta, \theta) = \frac{m}{2}(r^2 + r^2\theta^2) + \frac{\mu m}{r}.$ Then

(a) The generalized momenta of the system are $p_r = mr$ and $p_0 = mr^2\theta$

(b) The hamiltonian of the system is

$$H = \frac{1}{2m}\left[p_r^2 + \frac{p^2\theta}{r^2}\right] - \frac{1}{2}\frac{\mu m}{r}$$

(c) The hamiltonian of the system is

$$H = \frac{1}{2m}\left[p_r^2 + \frac{p^2\theta}{r^2}\right] - \frac{\mu m}{r}$$

(d) The generalized momenta of the system are $p_r = +mr$ and $p_\theta = -mr^2\theta$

95. Let u (x, y) be the solution of the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,$ which tends to zero as y $\to \infty$ and has the value sin x when y = 0. Then

(a) $u = \sum_{n=1}^{\infty} a_n \sin(nx + b_n)e^{-ny}$, where a_n are arbitrary and b_n are non-zero constant

(b) $u = \sum_{n=1}^{\infty} a_n \sin(nx + b_n)e^{-n^2 y}$, where $a_1 = 1$ and $a_n (n > 1), b_n$ are non-zero constants.

(c) $u = \sum_{n=1}^{\infty} a_n \sin(nx + b_n)e^{-ny}$ where $a_1 = 1,\ a_n = 0$ for $n > 1$ and $b_n = 0$ for $n \geq 1$.

(d) $u = \sum_{n=1}^{\infty} a_n \sin(nx + b_n)e^{-n^2 y}$ where $b_n = 0$ for $n \geq 0$ and a_n are all nonzero.

96. Let $f(x) = \sqrt{x+3}$ for $x \geq -3$. Consider the iteration $x_{n+1} = f(x_n),\ x_0 = 0\ ;\ n \geq 0$. The possible limits of the iteration are

(a) -1 (b) 3

(c) 0 (d) $\sqrt{3+\sqrt{3+\sqrt{3+...}}}$

97. Consider the boundary value problem

$$-u''(x) = \pi^2 u(x)\ ;\ x \in (0,1)\ \ u(0) = u(1) = 0.$$

If u and u' are continuous on [0, 1], then

(a) $u'^2(x) + \pi^2 u^2(x) = u'^2(0)$

(b) $\int_0^1 u'^2(x)dx - \pi^2 \int_0^1 u^2(x)dx = 0$

(c) $u'^2(x) + \pi^2 u^2(x) = 0$

(d) $\int_0^1 u'^2(x)dx - \pi^2 \int_0^1 u^2(x)dx = u'^2(0)$

98. To shown the existence of a minimizer for the functional $J[y] = \int_a^b f(x,y,y')dx,$ for which there is a minimizing sequence (φ_n), it is enough to have

(a) (φ_n) is convergent and J is continuous

(b) (φ_n) is convergent and J is differentiable

(c) (φ_n) has a convergent subsequence and J is continuous.

(d) (φ_n) has a convergent subseqeunce and J is differentiable.

99. Let $u(x,\ t)$ satisfy the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; x \in (0,\ 2\pi),\ t > 0$$

$u(x, 0) = e^{i\omega x}$ for some $\omega \in \mathbb{R}$. Then

(a) $u(x,t) = e^{i\omega x} e^{i\omega t}$

(b) $u(x,t) = e^{i\omega x} e^{-i\omega t}$

(c) $u(x,t) = e^{i\omega x}\left(\frac{e^{i\omega x} + e^{-i\omega t}}{2}\right)$

(d) $u(x,t) = t + \frac{x^2}{2}$

100. A solution of the PDE

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 - u = 0$$ represetns

(a) an ellipse in the x-y plane
(b) an ellipsoid in the xyz space
(c) a parabola in the u-x palne
(d) a hyperbola in the u-y plane

101. The iteration

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right),\ n \geq 0$$

for a given $x_0 \neq 0$ is an instance of

(a) fixed point iteration for $f(x) = x^2 - 2$

(b) Newton's method for $f(x) = x^2 - 2$

(c) fixed point iteration for $f(x) = \frac{x^2+2}{2x}$

(d) Newton's method for $f(x) = x^2 + 2$

102. Let $\lambda_1,\ \lambda_2$ be the characteristic numbers and $f_1,\ f_2$ be the corresponding eigen functions for the homogeneous integral equation

$$\varphi(x) - \lambda\int_0^1 (2xt + 4x^2)\varphi(t)dt = 0.$$ Then

(a) $\lambda_1 \neq \lambda_2$

(b) $\lambda_1 = \lambda_2$

(c) $\int_0^1 f_1(x) f_2(x)dx = 0$

(d) $\int_0^1 f_1(x) f_2(x)dx = 1$

103. Let $(X_n)_{n\geq 0}$ be a Markov chain on the state space $S := \{1, 2, \ldots, 23\}$ with transition probability given by

$$p_{i,i+1} = p_{i,i-1} = \frac{1}{2}\ \ \forall 2 \leq i \leq 22$$

$$p_{1,2} = p_{1,2,3} = \frac{1}{2}$$

$$p_{2,3,1} = p_{23,22} = \frac{1}{2}$$

Then, which of the following statements are true?

(a) $(X_n)_{n\geq 0}$ has a unique stationary distribution.

(b) $(X_n)_{n\geq 0}$ is irreducible

(c) $\mathbb{R}(X_n = 1) \to \frac{1}{23}$

(d) $(X_n)_{n\geq 0}$ is recurrent.

104. A fair coin tossed repeatedly. Let X be the number of fails before the first Head occurs. Let Y denote the number of Tails observed between the occurrence of the first and the second Head Let X+Y =N. Then which of the following statements are true?

(a) X and Y are independent random variables with

$$P(X = k) = P(Y = k) = \begin{cases} 2^{-(k+1)} & \text{for } k = 0,1,2,\ldots \\ 0 & \text{otherwise} \end{cases}$$

(b) N has probability mass function given by

$$P(N = k) = \begin{cases} (k-1)2^{-k} & \text{for } k = 2,3,4,\ldots \\ 0 & \text{otherwise} \end{cases}$$

(c) Given N = n, the conditional distribution of X and Y are independent

(d) Given N = n,

$$P\{X = k\} = \begin{cases} \frac{1}{n+1} & \text{for } k = 0,1,2,\ldots,n \\ 0 & \text{otherwise.} \end{cases}$$

105. Suppose that (X, Y) has a joint distribution with the marginal distribution of X being $N(0,\ 1)$ and $E(Y \mid X = x) = x^3$ for all $x \in \mathbb{R}$. Then which of the

following statements are true?

(a) Corr(X,Y) = 0.

(b) Corr(X, Y) > 0.

(c) Corr(X, Y) < 0.

(d) X and Y are independent.

106. An urn has 3 red and 6 black balls. Balls are drawn at random one by one without replacement. The probability that second red ball appers at the, fifth drawn is

(a) $\frac{1}{9!}$ (b) $\frac{4!}{9!}$

(c) $4\left(\frac{6!4!}{9!}\right)$ (d) $\frac{6!4!}{9!}$

107. Let $X_1,\ X_2 ..., X_n$ denote a random sample from a distribution having a probability density function $f(x\ ;\ \theta) = \theta\ x^{\theta-1},)\ 0 < x < 1$ zero elsewhere ; $\theta > 0$.

The set $\{(x_1, x_2,..., x_n) : \sum_1^n \log(x_i) \geq c\}$ where c is a suitably chosen real number, is a uniformly most powerful region for testing H_0 against H_1 when

(a) $H_0 : \theta = 1$ against $H_1 : \theta > 1$.

(b) $H_0 : \theta = 1$ against $H_1 : \theta \geq 4$.

(c) $H_0 : \theta = 4$ against $H_1 : \theta \leq 1$.

(d) $H_0 : \theta = 4$ against $H_1 : \theta \neq 1$.

108. Let $X_1,\ X_2,...$ be independent and identically distributed, each having a uniform distribution on (0, 1). Let $S_n \sum_{i=1}^n X_i$ for $n \geq 1$. Then, which of the following statements are true?

(a) $\frac{S_n}{n \log n} \to 0$ as $n \to \infty$ with probability 1.

(b) $P\left\{\left\{S_n > \frac{2n}{n}\right\} \text{occurs for infinitely many n}\right\} = 1$

(c) $\frac{S_n}{\log n} \to 0$ as $n \to \infty$ with probability 1

(d) $P\left\{\left\{S_n > \frac{n}{3}\right\} \text{occurs for infinitely many n}\right\} = 1.$

109. Suppose $\binom{X}{Y}$ is a random vector such that the marginal distribution of X and the marginal distribution of Y are the same and each is mormally distributed with mean 0 and variance 1. Then which of the following conditions imply independence of X and Y?

(a) Cov (X, Y) = 0

(b) aX+bY is normally distributed with mean 0 and variance $a^2 + b^2$ for all real a and b.

(c) $P(X \leq 0, Y \leq 0) = 1/4$

(d) $E[e^{itX+isY}] = E\left[e^{itX}\right] E[e^{isY}]$ for all real s and t.

110. A 2^4 experiment involving factors F_1, F_2, F_3 and F_4, each at two levels coded 0 and 1 is conducted in blocks of size 4 each. The block contents are as below:

Block I

F_1	F_2	F_3	F_4
0	0	0	0
0	1	1	0
1	0	1	1
1	1	0	1

Block II

F_1	F_2	F_3	F_4
0	0	0	1
0	1	1	1
1	0	1	0
1	1	0	0

Block III

F_1	F_2	F_3	F_4
0	0	1	0
0	1	0	0
1	0	0	1
1	1	1	1

Block IV

F_1	F_2	F_3	F_4
0	0	1	1
0	1	0	1
1	0	0	0
1	1	1	0

Then which of the following statements are true?

1. The confounded effects are $F_1F_2F_3, F_1F_2F_4, F_3F_4.$

(b) The confounded effects are $F_1F_2F_3, F_2F_3F_4, F_1F_4.$

(c) The desing is connected .

(d) The desing is disconnected.

111. $X_1, X_2,..., X_n$ are independently and identically distributed random variables, which follow $Bin(1,\ p)$.

To test $H_0 : p\frac{1}{2}$ vs $H_A : p\frac{3}{4}$, with size $\alpha = 0.01$ consider the test

$$\phi = \begin{cases} 1 & if \quad \sum_{i=1}^n X_i > c_n \\ 0 & \text{otherwise} \end{cases}$$

then which of the following statements are true?

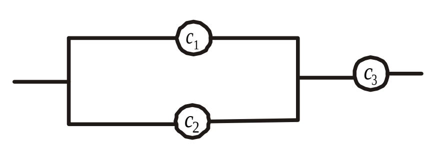
(a) As $n \to \infty$ power of the test converges to $\frac{1}{4}$.

(b) As $n \to \infty$ power of the test converges to $\frac{1}{2}$

(c) As $n \to \infty$ power of the test converges to $\frac{3}{4}$

(d) As $n \to \infty$ power of the test converges to 1.

112. A system consists of 3 components arranged as in the figure below:



Each of the components C_1, C_2, C_3 has independent and identically distributed lifetimes whose distribution is exponential with mean 1. Then the survival function, S(t), of the system is given by

(a) $S(t) = e^{-3t}$, for t > 0.

(b) $S(t) = (1-e^{-1})^2 e^{-t}$, for t > 0

(c) $S(t) = (1-e^{-2t})e^{-t}$, for t > 0

(d) $S(t) = (1-(1-e^{-t})^2)e^{-t}$, for t > 0

113. Let $X_1,...,X_n$ be independent and identically distributed random variables with $N(\mu,1)$ distribution. Assume that $\mu \in [0, \infty)$. Let $\hat{\mu}$ be the MLE of μ Then which of the following statements are true?

(a) $\hat{\mu} = \max(\bar{X}_n, 0)$.

(b) $\hat{\mu}$ is unbaised for μ.

(c) $\bar{X}_n$ is sufficient for μ.

(d) $\hat{\mu}$ is a consistent estimator fo μ.

114. Let $X_1, X_2,..., X_n$ be a random sample from $U(\theta, \theta+1)$. If $X_{(1)} < X_{(2)} < ... < X_{(n)}$ denote the ordered value of $X_1, X_2,..., X_n$, then which of the following statements are true?

(a) $(X_{(1)}, X_{(n)}+1)$ is joinly sufficient statistic for θ

(b) $X_{(n)}+1$ is a sufficinet statistic for θ

(c) $(X_{(1)}, X_{(n)})$ is a joinly sufficinet statistic for θ

(d) $X_{(1)}$ is a sufficient statistic for θ

115. A finite papulation has N units, labelled $U_1, U_2,..., U_n$ and the value of a study variable on unit U_i is $Y_i (i = 1, 2, ..., N)$. Let $Y = \sum_{i=1}^{N} Y_i$ and $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$, A sample of size $n > 1$ is drwan fron the papulation with probability proportational to size with replacement with selection prabablities $p_1, p_2,..., p_N; 0 < p_i < 1, i = 1, 2,..., N$ and $\sum_{i=1}^{N} p_i = 1$. Define $T = \frac{1}{n}\sum_{i \in s} Y_i / p_i$ where the sum extends over the units in the sample. Then, which of the following statements are true?

(a) T is an unbaised estimator of $\bar{Y}$?

(b) T is an unbaised estimator of Y

(c) The variance of T is zero if Y_i is proportional to p_i for all $i, i = 1, 2,..., N$.

(d) An unbaised estimator of the variance of T is $\frac{1}{n(n-1)}\sum_{i \in s}\left(\frac{Y_i}{p_i} - T\right)^2$

116. Let $Y_1, Y_2,..., Y_n$ be random variable with common unknown mean θ. The variance covariance matrix V of the vector $(Y_1, Y_2,.., Y_n)$ is such that the inverse of V has all its diagonal elements equal to c and all its off-diagonal elements equal to d. Let T_1 be the best linear unbaised estimator of θ and T_2 be the ordinary least squares estimator of θ. Which of the following statement are true?

1. $T_1 = \frac{1}{n}\sum_{i=1}^{n} Y_i = T_2$

2. $T_2 = n\bar{Y}$ and $T_1 = \sum_{i=1}^{n} Y_i - \bar{Y}$ where $\bar{Y}$ is the mean of the Y_i is

3. There are exactly $(n-1)$ linearly independent linear functions of $Y_1, Y_2, ..., Y_n$ each with zero expectation

4. There are exactly $(n\text{-}2)$ linearly independent linear functions of $Y_i, Y_2,..., Y_n$ each with zero expectation

117. Consider an M/MI queue with arrivals as a Poisson process at a rate of 8 per hour and a service time which is exponentially distributed at a rate of 6 minutes per customer. The waiting time of a customer in the queue

1. has a gamma distribution with p.d.f

$$f(x) = \begin{cases} \frac{(10)^8 x^7 e^{-10x}}{7!} & for\ x > 0. \end{cases}$$

2. has distribution function given by

$$F(x) = \begin{cases} 1-(0.8)e^{-2x} & for\ x>0 \\ 0 & otherwie. \end{cases}$$

3. has mean 4 minutes.

4. has mean 24 minutes .

118. Let X be a 4×1 random vector with Multivariate normal distribution with mean μ and dispersion matrix Σ . Suppose, the eigenvalue of Σ are $\lambda_1 = 6,\ \lambda_2, = 3,\ \lambda_3 = 2.,\ \lambda_4 = 1.$ Let $Y_1, Y_2, Y_3 Y_{,4}$ be the four principal components. Which of the following statements are correct?.

1. The percentage of variation explained by the first two components is $\leq 95\%$

2. The percetange of variation expalined by the first three components is $\geq 95\%$

3. Y_1, Y_2, Y_3, Y_4 are independent

4. Y_1, Y_2, Y_3, Y_4 have identical distribution.

119. Consider a region R, which is a triangle with vertices (0,0) , (0, θ), (0, θ). where $\theta > 0$. A sample of size *n* is selected at random from this region R. Denote the sample as $\{(X_i, Y_i): i = 1, 2, ..., n\}$ Then denoting $X_{(n)} = \max(X_1, X_2, ..., X_n)$ and $Y_{(n)} = \max(Y_1, Y_2, ..., Y_n)$ which of the following statemetns are true?

1. $X_{(n)}$ and $Y_{(n)}$ and independent
2. MLE of θ is $\frac{X_{(n)}+Y_{(n)}}{2}$
3. MLE of θ $\max_{1\le i\le n}\left(X_i+Y_i\right)$
4. MLE of θ is max $\{X_{(n)}, Y_{(n)}\}$

120. Let $X = (X_1, X_2, X_3, X_4)'$ be 4×1 random vector such that $X \sim N_4(O, \Sigma)$ wher $\Sigma = \begin{pmatrix} 1 & \rho & \rho & \rho \\ \rho & 1 & \rho & \rho \\ \rho` & \rho & 1 & \rho \\ \rho & \rho & \rho & 1 \end{pmatrix}$

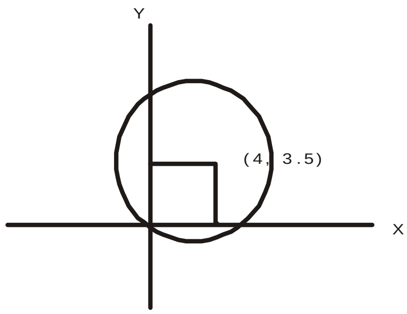
is positive define. Then which of the folowing statemetns are true?

1. $X_1X_2,\ X_2X_3$ and have identical distribution
2. $\frac{\left(X_1 - X_2\right)^2}{\left(X_1 - X_3\right)^2} \sim F_{1,1}$
3. $\{(X_1 - X_3)^2 + (X_2 - X_4)^2\}.\frac{1}{2(1-\rho)} \sim \chi_2^2.$
4. $\frac{\left(X_1 - X_2\right)^2}{\left(X_3 - X_4\right)^2} \sim F_{1,1}$

CSIR DECEMBER 2015 SOLUTION

PART 'A'

1.(4)



As perpendicular bisector of the chord of a circl passes
through the centre of the circle, hence centre will be

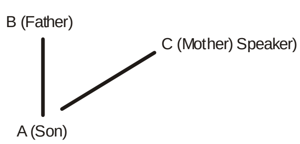
$\left(\frac{8}{2}, \frac{7}{2}\right)$ i.e, $(4,\ 3.5)$

2.(1)

Probability of being caught in atleast one trip =1 – probability being canght in no trip

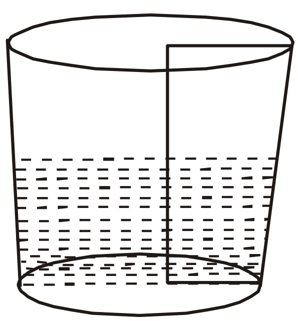
$= 1-(1-0.1)^4 = 1-(0.9)^4$

3.(2)



Statement is true in only one type of relation.

4.(4)



The ratio of empty to filled volume of the glass

$= \dfrac{\text{volume of empty frustum}}{\text{volume of filled trustum}}$

$= \dfrac{\frac{1}{3}\pi r^2 h\left(10^3-9^3\right)}{\frac{1}{3}\pi r^2 h\left(9^3-8^3\right)} = \dfrac{10^3-9^3}{9^3-8^3}$

where r and h are radius and height of the cone from which glass been extracted.

5.(2)

Number of diagonals of a convex polygon of n sides $= \dfrac{n(n-3)}{2}$ so for deodecagon (12 gon), number of diagonals $= \dfrac{12(12-3)}{2} = 54$

6.(1)

A plane can be filled with the help of tiles which are in regular hexagonal shape.

7.(1)

Total number of professors $= \dfrac{21+27+30}{2} = \dfrac{78}{2} = 39$

Number of professors who attended both th chennai and delhi meetings = $39-21 = 18$

8.(3)

Distance covered by inner and outer wheel is $\pi.1$ (Path is semicircular and there is unit spacing)

9.(1)

Each number is arithmetic mean of the numbers in next level between which it lies

So, $\dfrac{9+z}{2} = = -5 \Rightarrow z = -19$.

10.(4)

Letter wise we are capable of finding implicit statement as "THIS PROBLEM IS SOLVABLE BY INTEL LIGENT STUDENTS"

11.(3)

Graph in option (3) correctly represents distance versus frequency relation

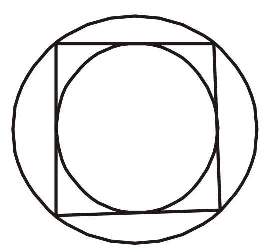
12.(4)

$3^{16} = 43046721$ which has 8 digits

13.(4)

Most Indian tropical fruit trees produce fruits in April -May because the impending monsoon provides optimum conditions for propagation.

14.(2)



diameter of outer circle = diagonal of square. Also, side of square =diameter of inner circle So, if r is the radius of inner circle $r\sqrt{2}$ will be radius of outercircle. So, Ratio of area of outer and inner circle will be

$\dfrac{\pi\left(r\sqrt{2}\right)^2}{\pi(r)^2} = 2$

15.(3)

As cos$x$ is monotonically decreasing in $\left(0,\ \pi/2\right)$ so.
cos r < cos d < cos g.

16.(2)

If premium price is x, then $8x+4\left(x/2\right) = 12\times(100+20)$

$\Rightarrow 10x = 1440 \Rightarrow x = 144$

17.(3)

Minimum numbers of lines required for the required purpose is 5.

18.(4)

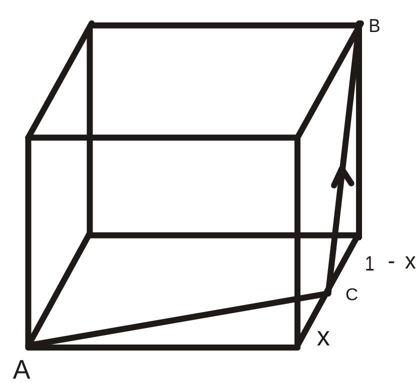
One of the ten pillars mean IX numbered pillar.

19.(2)

$\left.\begin{matrix} R & B & G \\ B & G & R \\ G & R & B \end{matrix}\right\}$ Boxes Balls

There are only two possibilities

20.(3)



Total distance covered from A to B along AC and then CB will be AC+CB = $\sqrt{1+x^2}+\sqrt{1+(1-x)^2}$ which will be minimum when $x = 1-x \Rightarrow x = 1/2$ So, minimum

distance $= \sqrt{1+\left(\frac{1}{2}\right)^2}+\sqrt{1+\left(1-\frac{1}{2}\right)^2} = \frac{\sqrt{5}}{2}+\frac{\sqrt{5}}{2} = \sqrt{5}$

PART 'B'

21.(4)

As sum of the eigen values of A is equal to trace of A, so, 5^{th} eigen value of A is $15-(2\times2+2\times3) = 5$
Now as determinant of A is product of eigen values of A, so $\det(A) = 2\times2\times3\times3\times5 = 180$

22.(3)

By Rolle's theorem there exist $C \in (0, 1)$ such that

$f'(c) = 0$
Now for Rolle's theorem on $f'(x)$ there exist a point $d \in (0, c)$ such that $f''(d) = 0$ $d \in (0, c) \Rightarrow d \in (0, 1)$.

So, $f''(x) = 0$ for some $x \in (0, 1)$

23.(1)

$$S_n = \sum_{k=1}^{n}\frac{1}{k} = 1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}$$

$$\Rightarrow S_{2^n} = \underbrace{1+\frac{1}{2}}_{>\frac{1}{2}} + \underbrace{\frac{1}{3}+\frac{1}{4}}_{>2.\frac{1}{4}} + \underbrace{\frac{1}{5}+...+\frac{1}{8}}_{>.\frac{4}{8}} + \underbrace{\frac{1}{2^{n-1}+1}+...+\frac{1}{2^n}}_{>\frac{2^{n-1}}{2^n}}$$

on R.H.S. there are n blocks each of which sums to a value greater than1/2. So, $S_{2^n} \geq \frac{n}{2}$ for every $n \geq 1$.

24.(2)

$T(p(x)) = p(x^2)$, $p(x) = 1 \Rightarrow p(x^2) = 1$

so, $T(1) = 1$

$p(x) = x \Rightarrow p(x^2) = x^2$

so, $T(x) = x^2$

$p(x) = x^2 \Rightarrow p(x^2) = (x^2)^2$

so, $T(x^2) = x^4$

$T(a+bx+cx^2) = a+bx^2+cx^4$

$T(a+bx+cx^2) = 0 \Rightarrow a+bx^2+cx^4 = 0$

$\Rightarrow a = b = c = 0$

So, Nullity (T) = 0

& Rank (T) = dim $\left(p_2(R)\right) - 0 = 3-0 = 3$

so, dim Range (T) = 3

25.(1)

Rank $\left(A_{3\times4}\right) = 2$, so A has 2 linearly independent rows (columns).Hence A^tA will be reduced to

$\begin{pmatrix} I_2 & 0 \\ 0 & 0 \end{pmatrix}$ by suitable elementary transformations

.Hence Rank of $A^tA = 2$

26.(4)

$$A = \begin{bmatrix} 91 & 31 & 0 \\ 29 & 31 & 0 \\ 79 & 23 & 59 \end{bmatrix}$$

$\Rightarrow$ determinant of A , det A or

$|A| = 59\times31(91-29) = 59\times31\times62 = 2\times(31)^2\times59$

$\Rightarrow$ det (A) $\neq 0$ if P is any prime number other than 2, 31 or 59 . so, S is infinite set.

27.(2)

According to the fixed point theorem for functions continuous in [a, b] it can be applied to function f from $[0, \infty]$ to $[0, \infty]$ if $f(x)$ is bounded because in this case if M is an upper bound then $f(x)$ is continuous in $[0, M]$ and codomain is also [0, M]

28.(4)

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$ By congruent operations

$R_3 \leftarrow -\frac{1}{2}R_3$ & $C_3 \leftarrow -\frac{1}{2}c_3$ we get A as congruent to

$$B = P^T AP = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1/2 \\ 0 & 0 & -1/2 & 0 \end{bmatrix}$$ whichhas

correspondingquadratic form as $Q(PU)=x^2+y^2-ZW$

29.(4)

$A^2 = A \Rightarrow$ A is idempotent $\Rightarrow I_n - A$ is also idempotent So, $(I_n - A)^2 = (I_n - A)$. Trace $(A) = K$ $\Rightarrow$ A has K eigenvalues 1, so n-K eigenvalues are 0. As idempotent matrices are diagonalisable so. $G.M$ of $0 = n$-$K = n$ - Rank (A) . Rank (A) = K = Trace (A) Also, Rank (A) + Rank $(I_n - A) = n$. As, $A \neq I_n$ so some eigenvalues of A are 0.

30.(3)

$\theta \in (-\pi, \pi)$ $\theta : R^2 \to (-\pi, \pi)$ such that $\theta(x, y) = \theta$

As codomain is bounded so the function is bounded, but limit at no point exist.

31.(2)

$$\lim_{n\to\infty} \frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{2}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{6}} + \ldots + \frac{1}{\sqrt{2n}+\sqrt{2n+2}}\right)$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{n}}\left(\frac{1}{2}\right)\left[\left(\sqrt{4}-\sqrt{2}\right)+\left(\sqrt{6}-\sqrt{4}\right)+\ldots\left(\sqrt{2n+2}-\sqrt{2n}\right)\right]$$

$$= \lim_{n\to\infty} \frac{1}{2\sqrt{n}}\left[\sqrt{2n+2}-\sqrt{2}\right] = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

32.(4)

The only subsets of R which are both closed aswell as open sets are R and ϕ . So if A is closed subset of R and $A \neq \phi$, $A \neq R$ then A will not be open

33.(2)

$x^3 = y^2 = (xy)^2 = 1$ implies that group G containing x and y will contain $\{1, x, x^2, y, xy, x^2y\}$. So, order of G is 6

34.(1)

$$T(z) = \frac{az+b}{cz+d} = \frac{(ax+b)+i(ay)}{(cx+d)+i(cy)}$$

$$= \frac{[(ax+b)+i(ay)][(cx+d)-i(cy)]}{(cx+d)^2+(cy)^2}$$

$$= \frac{(ax+b)(cx+d)+acy^2+i(ad-bc)y}{(cx+d)^2+(cy)^2}$$

$$\Rightarrow V = \frac{(ad-bc)}{(cx+d)^2+(cy)^2}.y$$

Now. $y > 0 \Rightarrow V > 0$ & $y < 0 \Rightarrow V > 0$

So, T maps H_+ to H_+ and H_- to H_-

35.(4)

$(x_1+x_2+x_3)(y_1+y_2+y_3+y_4) = 15 = 3\times 5$

$\Rightarrow x_1 = x_2 = x_3 = 1$ & $y_1+y_2+y_3+y_4 = 5$

So, any one y_i is equal to 2 and remaining y_i equals to 1. either $y_1 = 2$ or $y_2 = 2$ or $y_3 = 2$ or $y_4 = 2$ so there are 4 different solutions.

36.(3)

$x^{12}-1 = \left(x^4-1\right)\left(x^8+x^4+1\right)$

$= \left(x^4-1\right)\left(x^4+x^2+1\right)\left(x^4-x^2+1\right)$

$= (x-1)(x+1)(x^2+1)(x^4+x^2+1)(x^4-x^2+1)$ which are irreducible factors.

37. 3

$S = \{z \in C | z^{98} = 1$ & $z^n \neq 1,\ 0 < n < 98\}$

$98 = 2\times 7^2$

$Z^{98} = 1 \Rightarrow Z = (1)^{1/98}$

i.e., 98^{th} roots of unity

$\Rightarrow Z = 1, e^{\frac{i2\pi}{98}}, e^{\frac{i4\pi}{98}}, \ldots, e^{\frac{i194\pi}{98}}$

As, $Z^n = 1$ only for those multiples of 2π which are multiples of 2 or 7 from 0, 1, 2, ..., 97

No.	of	M(2) = 49
"	"	M(7) = 14
"	"	M(14) = 7

No. of M (2 or 7) = 49 + 14 - 7 = 56.So, |S| = 98 - 56 = 42

38.(2)

$\hat{A} = \hat{\hat{A}}$, as $\hat{A}$ is union of A and all relatively compact connected components of X \ A.

39.(3)

R[X] i.e., set of all polynomials in x with real coefficie-ents is unique factorization domain but not a princip al ideal domain.

40.(2)

$f(z) = \sum_{n=1}^{\infty} n \log n\, z^n$ $a_n = n\log n$

$n \le a_n \le n^2 \Rightarrow n^{1/n} \le a_n^{1/n} \le (n^2)^{1/n}$

$\Rightarrow \lim n^{1/n} \le \Rightarrow \lim a_n^{1/n} \le \lim\left(n^{1/n}\right)^2$

$$\Rightarrow 1 \le \lim a_n^{1/n} \le 1 \Rightarrow \lim a_n^{1/n} = 1$$

$$g(z) = \sum_{n=1}^{\infty} \frac{e^{n^2}}{n} z^n \;,\; a_n = \frac{e^{n^2}}{n} \Rightarrow a_n^{1/n} = \left(\frac{e^{n^2}}{n}\right)^{1/n}$$

$$\lim_{n\to\infty} a_n^{1/n} = \lim_{n\to\infty} \frac{e^n}{n^{1/n}} \to \infty$$ So, radius of convergence

$$R = \frac{1}{\infty} = 0$$

41.(2)

$\vec{F} = 5\hat{i} - 2\hat{j} + 3\hat{k}, \vec{r} = 2\hat{i} + \hat{j} - 2\hat{k}$, Torque = $\hat{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 5 & -2 & 3 \end{vmatrix} = -\hat{i} - 16\hat{j} - 9\hat{k}$$

42.(2)

$$F(x, y, y') = y^2 + y'^2 - 2y \, Sin \, x \quad \frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$$

$$\Rightarrow 2y - Sin x - \frac{d}{dx}(2y') = 0 \Rightarrow 2(y'' - y) = Sin \, x$$

$\Rightarrow y'' - y = -\frac{1}{2} Sin \, x$. C.F. is $y = c_1 e^x + c_2 e^{-x}$

P.I. is $y = \frac{1}{2}\sin x$ So, Extremal is

$$y = C_1 e^x + C_2 e^{-x} + \frac{1}{2} Sin \, x$$

43.(2)

$$R(x, t, \lambda) = \sum_{r=0}^{\infty} \lambda^r k_r(t)$$

$$k_0(t) = 1 \; ; \; k_1(t) = \int_t^x z dz = (x-t)$$

$$k_2(t) = \int_t^x (z-t) dz = \frac{(x-t)^2}{2} \; \ldots \; k_r(t) = \frac{(x-t)^r}{r!}$$

$$\Rightarrow R(x, t, \lambda) = \sum_{r=0}^{\infty} \lambda^r \frac{(x-t)^r}{r!}$$

$$\sum_{r=0}^{\infty} \frac{[\lambda(x-t)]^r}{r!} = e^{\lambda(x-t)}$$

44.(2)

A.E. is

$$\frac{dx}{x-y} = \frac{dy}{y-x-u} = \frac{du}{u}$$

$$\Rightarrow \frac{dx+dy+du}{0} = \frac{du}{u}$$

$$\Rightarrow dx+dy+du = 0 \Rightarrow (x+y+u) = c_1$$

$$u(x, 0) = 1, \Rightarrow c_1 = x+1$$

From these conditions solution of IVP satisfies

$u^2(x+y+u)+(y-x-u) = 0$ ....(1)

$u(x, 0) = 1$ in eq. 1 gives $L.H.S = 1(x+0+1)+(0-x-1)$

$= x+1-x-1 = 0 = R.H.S.$

45.(3)

$$f(x) = ax+100 \Rightarrow f'(x) = a$$

$x_{n+1} = f(x_n)$ will converge if $|f'(x)| < 1 \Rightarrow |a| < 1$

So, a = 0.1 is possible from given options

46.(4)

$u(x, y) = \frac{x^3}{6} + C$ where C is any real number can become particular integral so there is more than one particular integral.

47.(1)

$$y'(x) = f(y(x))$$

$$f(y(x)) = y'(-x)$$

$$\Rightarrow f(y(-x)) = y'(-x)$$

$$\Rightarrow f(-y(x) = f(y(x)) = -y'(x)$$

$$\Rightarrow y'(x) = -f(y(x))$$

$$\Rightarrow y(-x)$$

is not a solution but $-y(-x)$ is a solution

48.(4)

$$\frac{dY}{dt} = AY = \begin{bmatrix} \frac{dy_1}{dt} \\ \frac{dy_2}{dt} \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$\Rightarrow \frac{dy_1}{dt} = -y_1 + y_2 \;\&\; \frac{dy_2}{dt} = -y_2$$

$$\frac{dy_2}{y_2} = dt \Rightarrow ln \, y_2 = -t+k$$

$$\Rightarrow y_2 = c_1 e^{-t} \; ; \; y_2(0) = 1 \Rightarrow c_1 = 1 \Rightarrow y_2(t) = e^{-t}$$

Now, $\frac{dy_1}{dt} = -y_1 + e^{-t}$ A.E. is $m + 1 = 0 \Rightarrow m = -1$

C.F. is $y_1(t) = C_2 e^{-t}$

P.I. is $y_1(t) = \frac{1}{(D+1)} e^{-t} = te^{-t}$

$$\Rightarrow y_1(t) = C_2 e^{-t} + te^{-t}$$

$$y_1(0) = 0 \Rightarrow C_2 = 0$$

$$y_1'(t) = te^{-t}$$

$y_1'(t) = (1-t)\, e^{-t} < 0$, if $t > 1$

So, $y_1(t)$ is monotonically decreasing if t > 1.

$y_2'(t) = -e^{-t} < 0$ if t > 1 , So $y_2(t)$ is also monotonically decreasing if t > 1.

49.(4)

Required probability = $\frac{2\times 3!}{(6)^3} = \frac{12}{216} = \frac{1}{18}$

50.(3)

$F(x)+F(-x) = P(X \le x)+P(X \le -x) = 1+P(X = x)$

51.(4)

Mode will remain same

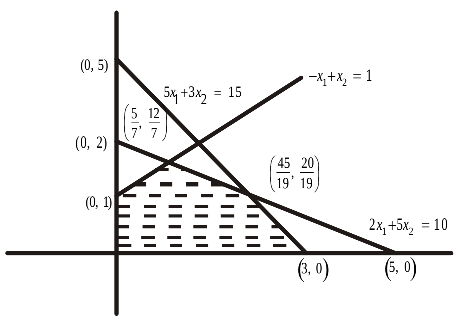
52.(2)

$UE(\sigma^2) = \frac{1}{2}\left(e_1^2+e_2^2\right)$

53.(2)

54.(3)

55.(2)



All the points lying on $\left(\frac{5}{7}, \frac{12}{7}\right)$ and $\left(\frac{45}{19}, \frac{20}{19}\right)$ are solutions so there are infinitely many solutions.

56.(4)

$P(X_1+X_2+...+X_n > n\log n)$ converges to 0.

57.(1) 58.(2) 59.(4) 60.(2)

PART 'C'

61.(1,2)

$\lim_{n\to\infty} P_n(\lambda_0) = \lambda_0^2 \ lt \ a_n+\lambda_0 \ ltb_n$

$\lim_{n\to\infty} P_n(\lambda_1) = \lambda_1^2 \ lt \ a_n+\lambda_1 \ lt \ b_n$

$\lambda_0^2 \ lt \ a_n+\lambda_0 \ lt \ b_n = l_1$ .....(1)

$\lambda_1^2 \ lt \ a_n+\lambda_1 \ lt \ b_n = l_2$ .....(2)

Equation (1) $\times \lambda_1$ and equation (2) $\times \lambda_2$ gives

$\lambda_1\lambda_0(\lambda_0-\lambda_1) \ lt \ a_n = \lambda_1 l_1-\lambda_0 l_1$

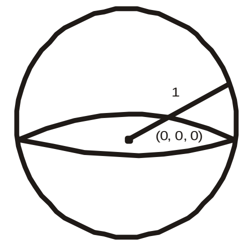
$\Rightarrow \ lt \ a_n = \frac{\lambda_1 l_1-\lambda_0 l_2}{\lambda_1\lambda_0(\lambda_0-\lambda_1)}$ (i.e exist)

$lt \ a_n$ exist and $\lambda_0^2 \ lt \ a_n+\lambda_0 \ lt \ b_n$ exist so $lt \ b_n$ exist

Thus $\lim_{n\to\infty} P_n(x) = x^2 \ lt \ a_n + x \ ltb_n$ which exist for all $x\in R$. Also, $P_n'(x) = 2a_n+b_n$

$\lim_{n\to\infty} P_n'(x) \ 2x \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n$ which also exist for all $x\in R$.

62.(1,3)



$x^2+y^2+z^2 = 1$ represents points on unit sphere which is closed as well as bounded. Also, $\prod_{n=1}^{\infty} A_n$ with product topology where A_n with two element 0 and 1 has discrete topology for n = 1, 2, 3, ... admits finite subcover for every cover, so it is compact.

63.(1, 2, 3, 4)

$\sup_{x\in R} |f'(x)| < \infty$ implies that modulus of $f'(x)$ i.e $f'(x)$ is bounded so $f(x)$ is uniformly continuous .Also, under uniformly continuous function f

(i) bounded sequence is mapped onto bounded sequence

(ii) Convergent sequence is mapped onto convergent sequence

(iii) Cauchy sequence is mapped onto cauchy sequence

64,(1,3)

Given series is $\lim_{n\to\infty} \sum_{\ell_1 k=0}^{n} \frac{k^2 x^k y^\ell}{\ell!} \ \forall y \in R \ \sum_{\ell=0}^{\infty} \frac{y^\ell}{\ell} = e^y$

i.e., the series is convergent. But if $|x| < 1$ then only $\sum x^k$ will be convergent iff $|x| < 1$ i.e, $x\in(-1, 1)$ so for $(x, y) = (-1, 1) \times (0, \infty)$ and $(-1, 1) \times (-1, 1)$ series is convergent

65.(3, 4)

$|f(x)-f(y)| \le |\cos x-\cos y|$

$\Rightarrow |f(x)-f(y)| \le |x-y|$. so $f(x)$ is Lipschitz function so, $f(x)$ is uniformly continuous on (0, 1) so also limit at either end points must exist. So,

$\lim_{x\to 0} f(x)$ exist

66.(2, 4)

In U if $x > 0$ or $x < 0$ which also is the sign of y, then there will be one- to- one correspondence between U and R^2 and also if principal value is taken so, options 2 & 4 are correct

67.(2, 4)

$S = \left\{\left(m+\frac{1}{4^{|p|}}, \ n+\frac{1}{4^{|q|}}\right) | m, \ n, \ p, \ q\in Z\right\}$

$\Rightarrow S^1$ Derived set of S $=\{(m\ n): m\ n\ \in\ Z\}$.Also,

S^c has polygonal path between any two points in it so it is path connected

68.(1, 2, 3)

$A=\{(x,\ y)\in R^2 : x+y\neq -1\}$

$f(x,\ y)\ =\ \left(\frac{y}{1+x+y},\ \frac{x}{1+x+y}\right)$,Jacobian of trasformation

$$J\ =\begin{pmatrix} -\frac{y}{(1+x+y)^2} & \frac{1+x}{(1+x+y)^2} \\ \frac{1+y}{(1+x+y)^2} & -\frac{x}{(1+x+y)^2} \end{pmatrix}$$

Det $(J) = |J| = -\frac{1}{(1+x+y)}$ which does not vanish on A. Also on A $f(x,\ y)$ is infinitely differentiable

Now. $f(x,\ y)\ =\ (a,\ b)$ gives

$y\ =\ a(1+x+y)\ \&\ x\ =\ b(1+x+y)$

$\Rightarrow (1-a)y-ax\ =\ a$

$-b\ y+(1-b)\ x\ =\ b$

$$\Rightarrow \begin{bmatrix} 1-a & -a \\ -b & 1-b \end{bmatrix}\begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

. Determinant of coefficient matrix is 1-a-b . So, if $a + b\neq 1$ system has unique solution . So, f is one-to-one on A . For $a + b = 1$ there cannot be $(x, y)\in A$ such that $f(x,\ y)\ =\ (a,\ b)$ so $f(A)$ is not equal to R^2

69.(3,4)

$|xy|$ is differentiable at (0, 0) so $f(x,\ y)$ is differentiable at (0, 0) , $Df(0,\ 0)\ =\ \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$,

$|Df(0,\ 0)| = 5\neq 0$ So, At (0, 0) $Df(0, 0)$ is also invertiable

70.(3, 4)

If $\{f_n\}$ is real valued continuous functions in $[0,\ \infty)$ and converges to f uniformly on $[0,\ \infty)$, then f is continuous on $[0,\ \infty)$

Now, if $f_n(x)=\frac{x}{n^2}$; $x<n$

$=0$ $otherwise$

and $x\in[0,\ \infty)$ Then $f(x)\ =\ 0$. Also $f_n(x)$ onverges uniformly to $f(x)$ but $\int_0^\infty f_n(x)dx\ =\ \int_0^n \frac{x}{n^2}dx\ =\frac{1}{2}$

and $\int_0^\infty\ f(x)dx\ =\ 0$, So, $\int_0^\infty f_n(x)\ \neq\ \int_0^\infty f(x)dx$

71. $x\ =\ (x_1,\ x_2,...,\ x_n)\ \Rightarrow tx\ =\ (tx_1,\ tx_2,\ ...,\ tx_n)$

Jacobian of transformation $=\ t^{-n}$

$\int_{B_a} f(tx)dx = \int_{B_{ta}} f(x)t^{-n}dt$ and also if $y=(0,0,...,0)$

$\int_{R^n} f(x+y)dx\ =\ \int_{R^n} f(x)dx$

72.(1, 2)

$f^{-1}(G_1\cup G_2)\ =\ f^{-1}(G_1)\cup f^{-1}(G_2)$

$f^{-1}(G_1^C)\ =\ f^{-1}(R^2\backslash G_1)$

$=\ f^{-1}(R^2)\backslash f^{-1}(G_1)$

$=\ R^2\backslash f^{-1}(G_1)\ =\ \left(f^{-1}(G_1)\right)^c$

Now. Let $f(x,\ y)\ =\ (x^2,\ y^2)$

$G_1\ =\ \{(1,\ 1)\}$; $G_2\ =\ \{(-1,\ -1)\}$

$\Rightarrow\ f(G_1\cap G_2)\ =\ f(\phi)\ =\ \phi$

$f(G_1)\ \cap\ f(G_2)\ =\ \{(1,\ 1)\}$ and infact

$f(G_1\cap G_2)\subseteq f(G_1)\cap f(G_2)$. Also, $G_1\ =\ \phi\ \&\ G_2\ =\ \phi$

implies G_1 is open and G_2 is closed then G_1+G_2 is both open and closed

73.(1, 3)

If A and B are square matrices of same order then AB and BA have same eigen values. Also A^{-1} exist then $A^{-1}(AB)\ A\ =\ BA$ so, AB and BA are similar matrices

74.(1, 2, 3, 4)

$\dim V\ =\ rank(T)+nullity(T)$ &

$dimV\ =\ rank(T^2)+nullity(T^2)$

Rank (T) = Rank (T^2)

$\Rightarrow$ Nullity (T) = Nullity (T^2)

As $T(\lambda)\ =\ 0\ \Rightarrow T^2(\lambda)\ =\ 0$

$\Rightarrow$ Null space of $T\subseteq$ Null space of T^2 But nullity (T) = Nullity (T^2)

$\Rightarrow$ Null space of T = Null sapce of T^2

So, $T^2(\lambda)\ =\ 0\Rightarrow T(\lambda)\ =\ 0$

So, kernel (T) and Rank (T) are disjoint

So, also kernel (T^2) and Rank (T^2) are disjoint

75.(1, 2, 3, 4)

$T(p(x))\ =\ 0\ \Rightarrow\ a_n+a_{n-1}x+...+a_0x^n\ =\ 0$

$\Rightarrow a_0\ =\ a_1\ =\ ...\ a_n\ =\ 0$

So, Nullity (T) = 0 and Rank (T) = n+1 . Hence T is one- to -one & So T is also invertible Transformation matrix T is

$$T = \begin{bmatrix} & 0 & 0 & 0 & \cdots & 0 & 1 \\ & 0 & 0 & 0 & \cdots & 1 & 0 \\ & \vdots & & & & & \\ 0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

det (T) = one or minus one = ± 1

76.(1, 3, 4)

G.M. of eigen value2 of A is 3Rank (A-2I) = 3 – 2 = 1 As A.M. $\neq$ G.M. for eigen value 2, hence A is not diagonalisable So, $C(x) = (x-2)^2(x-3)$ and $m(x) = (x-2)^2(x-3)$, so its Jordan canomical form is $B = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. As eigen values of A are from Q,

So A is similar to B over Q.

77.(2, 4)

If u and v are solutions of $Ax = b$ then $Au = b$ & $Av = b \Rightarrow A(\lambda u+(1-\lambda)v) = \lambda b(1-\lambda)b = b$ so $\lambda u+(1-\lambda)v$ is also solution of Ax = b for $\lambda \in R$. If Rank $A = n$ then then $Ax = b$ has unique solution which also means at most one solution .

78.(1, 3)

every non-zero vectors are eigen vectors of A, means $AX=\lambda X \Rightarrow (A-\lambda I)X=0$ has n linearl independent solutions, so rank of $A-\lambda I$ must be0 ;hence $A-\lambda I$ null matrix $\Rightarrow A = \lambda I$ for some $\lambda \in C \Rightarrow$ All eigen values of A are equal

79.(3, 4)

$f: R \to R^2$ givenby $f(x) = (\sin x, \cos x)$ is continuous map and also $f(R) = \left\{(x, y) \in R^2 \mid x^2+y^2 = 1\right\}$. Also,

$f : [0, 1] \cup [2, 3] \to \{0, 1\}$ given by

$$f(x) = 0 \quad if \quad x \in [0, 1$$
$$= 1 \quad if \quad x \in [2, 3]$$

is a continuous map from [0, 1] U [2, 3] to {0, 1}

80.(2, 4)

$X \equiv 2 (\text{mod } 5)$ & $X \equiv 3 (\text{mod } 7)$

$\Rightarrow X \equiv 17 (\text{mod } 35)$

$X \equiv 17 \ (\text{mod } 35)$ & $X \equiv 4 \ (\text{mod } 11)$

$\Rightarrow X \equiv 367 (\text{mod } 385)$

$\Rightarrow X = 367, 752, 1137, 1522, ...$

System has a solution in [601,800] & [1001,1200]

81.(1, 4)

For n = 5 disjoint cycles will be

$1, 4 \to 5_{C_1} \times 4_{C_4}(4-1)! = 30 \ \ way$

$2, 3 \to 5_{C_2} \times 3_{C_3} \times (2-1)! \times (3-1)! = 20 \ way \quad \Rightarrow a_5 = 50$

Total 50 ways

For n = 4 disjoint cycles will be 1,

$\Rightarrow a_4 = 11$

82, (1, 2, 4)

X^3 contains factors X, X^2 and X^3, So, there are 3 distinct proper ideals in $\frac{Q[X]}{X^3}$, but prime ideal is unique. A is not an integral domain

$\vec{f} \cdot \vec{g} = 0 \Rightarrow f(0)\, g(0) = 0$ as . $\vec{f} \cdot \vec{g} = 0$ at least one of them must be zero

83.(3,4)

$$W = Cos\frac{2\pi}{10} + i\ Sin\frac{2\pi}{10} = e^{-\pi/5}$$

$$W^2 = \left(e^{i2\pi/5}\right)^2 = e^{i4\pi/5}$$

Also $w^5 = 1$ *Set* $\{1, w, w^2, w^3, w^4\}$ and $\{1, w^2, w^4, w^6, w^8\}$ are same so, these set over Q will span $L = Q(w)$ & K= $Q(w^2)$ respectively Hence$L = K$

84.(1)

Number of sylow -5 subgroups = 1+ 5k where 1+ 5k divides 60 $\Rightarrow$ 1+ 5k = 1 or 6 . But as G is a simple group so, 1 cannot be the possibility hence G has six sylow - 5 subgroups

85.(3,4)

X^2+X+1 is irreducible over Q , so $\frac{Q[x]}{(x^2+x+1)}$ is field

also X^2+X+1 is irreducible over F_2 so $\frac{F_2[X]}{(X^2+X+1)}$ is also a field

86.(1, 2, 3, 4)

$a_n = \frac{1}{n} \Rightarrow \lim\ a_n = 0$ .So $f(z)$ is constant

$a_n = (-1)^{n-1}.\frac{1}{n} \Rightarrow \lim\ a_n = 0$. So, again $f(z)$ is constant $a_n = \frac{1}{2n} \Rightarrow \lim\ a_n = 0$ .So, $f(z)$ is again constant $a_n = n$ if 4 does not divide n , $= \frac{1}{n}$ is 4 divides n , has subsequence $a_{4n} = \frac{1}{4n}$ such that $\lim a_{4n} = 0,$ so, $f(z)$ is again constant

87.(1, 4)

163 is prime number so, $\left(\frac{Z}{163Z}\right)$ is cyclic also,

group of it's units is also cyclic units of $\left(Z/_{10Z}\right)$ = {1 , 3, 7, 9} which is cyclic group generated by {3} or {7}

88.(1, 2, 3, 4)

$f(z) = \frac{1}{e^z - 1}; e^z \neq 1$

Now $e^Z - 1 = 0 \Rightarrow e^Z = 1$

$\Rightarrow z = ln1 = ln1\ e^{i2n\pi}$

$\Rightarrow$ z = i2n π ; n = 0, $\pm$ 1 $\pm$ 2,...

Also $\lim\limits_{Z \to i2n\pi} \frac{z - i2n\pi}{e^Z - 1}$ $[\frac{0}{0} Case]$

$\lim\limits_{z \to i2n\pi} \frac{1}{e^z}$ [By L' Hospital rule} = 1 $\neq$ 0 so $f(x)$ has simple pole at $2n\pi i$. As, $f(z)$ has only singularities as pole, so $f(z)$ is homomorphic function Also all the poles are on imaginary axis

89.(2, 3)

$f(z) = \frac{1}{z}$ on annulus $A = \{z \in C : \frac{1}{2} < |z| < 2\}$ has laurent series expansion about the centre of the annulu whose pole is 0 lying out side A. Such approximatin sequence cannot be polynomial but it will be rational function only.

90.(4)

K = Compact subset of C with non-empty interior

$\|\bullet\|_K$ will be norm on vector sapce of entire function but not on vector sapce of continuous functions.

91.(4)

$y(t) = y(0) + \int_0^t y(s)ds$ ...(1)

Squaring both sides we get

$y^2(t) = y^2(0) + \left(\int_0^t y(s)ds\right)^2 + 2y(0)\int_0^t y(s)ds$

Differentiating both sides *of* equation (1) we get

$y'(t) = y(t) \Rightarrow y(t) = ce^t \Rightarrow y(t) = y(0)\ e^t$

Now. $\left(\int_0^t y(s)ds\right)^2 + 2y(0)\int_0^t y(s)ds$

$= y^2(0)(e^t - 1)^2 + 2y^2(0)(e^t - 1)$

$= y^2(0)(e^{2t} - 1)^2 + 2y^2(0)(\frac{e^{2t} - 1}{2}) = 2\int_0^t y^2(s)ds.$

So, $y^2(t) = y^2(0) + 2\int_0^t y^2(s)ds$

92.(3,4)

Both H and L are quadratic in V and also H = L

93.(1,2,3,4)

$u'(t) = 4u^{3/4}(t)$ ....(1)

$u(t) = 0 \Rightarrow u'(t) = 0$ equation - (1) is satisfied

$u(t) = t^4 \Rightarrow u'(t) = 4t^3$

$4u^{3/4}(t) = 4t^3$ So, $u(t) = t^4$ is also a solution

$u(t) = 0\ ; 0 < t < 1$
$\quad = (t-1)^4 ; t \geq 1$

$\Rightarrow u'(t) = 0\ ; 0 < t < 1$
$\quad = 4(t-1)^3\ ;\ t > 1$
$\quad = 0\ ; t = 1$

So, this $u(t)$ also satisfioes the given differential equation. similarly

$u(t) = 0\ ;\ 0 < t < 10$
$\quad = (t-10)^4\ ;\ t \leq 10$ has

$u'(t) = 0\ ;\ 0 < t \leq 10$
$\quad = 4(t-10)^3\ ;\ t > 10$

So this u(t) also satisfies given differential equation

94.(1,3)

95.(3)

For elliptic equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, solution $u(x, y)$ such that $\lim\limits_{y \to \infty} u(x, y) = 0$ and $u(x, y)\ \int_{y=0} =$ sin x will be $u(x, y) = \sum\limits_{n=1}^{\infty} a_n \sin(nx + b_n)e^{-ny}$

96. 4

$f(x) = \sqrt{x+3}$

$x_0 = 0\ ;\ x_{n+1} = f(x_n)$

$\Rightarrow x_1 = \sqrt{3}\ ; x_2 = \sqrt{3+\sqrt{3}}$

....point of convergence is $\sqrt{3+\sqrt{3+\sqrt{3+...}}}$

97. 1,2

$-u''(x) = \pi^2 u(x)$

$\Rightarrow u''(x) + \pi^2 u(x) = 0$

$\Rightarrow (D^2 + \pi^2)u = 0$

$\Rightarrow u(x) = A\ \cos\pi x + B\sin\pi x$

$u(0) = 0 \Rightarrow A = 0$
$u(1) = 0 \Rightarrow -A = 0$

$\Rightarrow u(x) = B\sin\pi x$
$\Rightarrow u'(x) = \pi B\cos\pi x$

$u'(0) = -B\pi$

So, $u'^2(x) + \pi^2 u^2(x) = \pi^2 b^2 = u'^2(0)$

Now, $\int_0^1\ \pi^2 B^2 \cos 2\pi x\ dx$

$= \int_0^{1/2} \pi^2 B^2 \cos 2\pi x\ dx = 0$

$= \int_0^1 u'^2(x)dx - \pi^2 \int_0^1 u^2(x)dx = 0$

98.(1,2,3,4)

For existence of a minimizer it is sufficient to have a convergent subsequence of $\{\phi_n\}$ and J is continuous .....(option 3)
Rest option 1, 2 & 4 are also sufficient because they imply option 3.

99.(1,2,3)

For the given hyperbolic equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ with $u(x,\ 0) = e^{i\omega x}$ by separtion of function $u(x,\ t) = e^{i\omega x}\ e^{i\omega t}$

$u(x,\ t) = e^{i\omega x}\ e^{-i\omega t}$ as well as $u(x,\ t) e^{i\omega x}\left(\frac{e^{i\omega t} + e^{-i\omega t}}{2}\right)$

Satisfies the given equation but for $u(x,\ t) = t + \frac{x^2}{2}$

$u(x,\ 0) = \frac{x^2}{2}$ so it does not satisfy the given condition

100.(3)

$u = px + qy + p^2 + q^2$. It is quadratic in p so solution will be parabolic in u- x plane

101.(2,3)

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right) = \frac{x_n^2 + 2}{2x_n}$$

$\Rightarrow x = f(x)$ where $f(x) = \frac{x^2+2}{2x}$ for fixed point iteration

Also, $f(x) = x^2 - 2 \Rightarrow f'(x) = 2x$ so, for .Newton's method iterative formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - 2}{2x_n}$$

$$= \frac{2x_n^2 - x_n^2 + 2}{2x_n} = \frac{x_n^2 + 2}{2x_n}$$

102. 1, 3

Characteristic numbers are distinct for the given homogeneous integral equation and the corresponding eigen functions are orthogonal to each so, $\lambda_1 \neq \lambda_2$ and $\int_0^1 f_1(x) f_2(x) dx = 0$

103.	1,2,3,4	104.	1,4	105.	2
106.	3	107.	1,2	108.	1,4
109.	2,4	110.	2,4	111.	4
112.	4	113.	1,3,4	114.	1,3
115.	2,3,4	116.	1,3	117.	2,4
118.	1,2,3	119.	3	120.	1,3,4